KIN 335 - Biomechanics

Impacts and the Coefficient of Restitution

Reading Assignment: Bernardo, S. (May, 1984). Physics of the sweet spot. *Science Digest*, pp. 62-65, 95; Bryant, F.O., Burkett, L.N., Chen, S.S., Krahenbuhl, G.S., & Lu, P. (1977). Dynamic and performance characteristics of baseball bats. *Research Quarterly*, **48** (3), 505-509. Associated Press (2002). Now soccer has a rabbit ball too [online].

Introduction and Theory: There are many sporting activities in which one object collides with another (e.g., a baseball bat and a pitched ball, tennis racquet and ball, golf club and ball, bowling ball and pins). These impacts generally occur over a very short period of time, involve contact forces of high magnitude, and result in rapid changes in momentum of one or both colliding objects. In addition, during the brief time of impact, the two colliding objects typically will undergo a period of deformation (i.e., change in shape) and a period of restitution (i.e., return to the original shape). Consider as an example the contact between a tennis racquet and ball. The ball is in contact with the racquet strings for only a few milliseconds. In the early portion of the impact, the ball flattens considerably and the strings are distorted. In the latter portion of the impact, both the ball and the racquet rapidly return toward their original shapes.

The velocities of the two colliding objects following the impact depend on their velocities before impact and the nature or "quality" of the impact. In a perfectly elastic collision, the relative velocities of the two objects after impact (separation velocities) are the same as their relative velocities before impact (approach velocities). In an inelastic collision, the relative velocities of the colliding objects after impact are less than those before impact and some of the total energy of motion is lost (e.g., some may be transformed into heat associated with the deformation and restitution processes).

The coefficient of restitution (e) is an index that describes the relative elasticity of an impact and ranges between 0 to 1. An e equal to 1 reflects a perfectly elastic collision, whereas an e equal to 0 reflects a perfectly plastic (or inelastic) collision. The coefficient of restitution depends to a large extent on the nature of the two materials of which the colliding objects are made. It is also affected by the impact velocity, the shape and size of the colliding objects, the location on the colliding objects at which the collision occurs, and their temperatures.

In quantitative terms, e is the ratio of the relative velocities of the colliding objects before and after impact:

$$e = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right) \qquad \textbf{(1)}$$

where v_1 and v_2 are the velocities of the two colliding objects immediately after impact and u_1 and u_2 are their velocities immediately before impact. (Note: e is always a positive number. The minus sign in the equation is needed because the relative velocities before and after impact are in the opposite directions.) For impacts of a ball or similar object (i.e., object 1) off a fixed surface such as the floor (object 2), equation 1 can be simplified to:

$$e = -\left(\frac{v_1}{u_1}\right) \qquad \textbf{(2)}$$

since the velocity of the floor before and after impact is, for all practical purses, zero (i.e., $u_2 = v_2 = 0$).

From our knowledge of uniformly accelerated motion, we know that when a ball is dropped onto a fixed surface, the velocity of the ball immediately before impact is determined by the height from which it is dropped ($v_f^2 = v_i^2 + 2ad$, where $v_i = 0$). It is clear from this equation of uniformly accelerated motion that the velocity is proportional to the square root of the height. Similarly, the height the ball reaches after impact is proportional to the square root of the velocity of the ball immediately after impact. Thus, equation 2 can be further simplified for this special case in which a ball is dropped onto a fixed surface:

$$e = \sqrt{\frac{h_b}{h_d}} \qquad \textbf{(3)}$$

where h_b and h_d are the rebound and drop heights, respectively.

Purpose: To examine the effect of the materials of which the colliding objects are made and the temperature of the colliding objects on the coefficient of restitution for impacts between a ball and the floor.

Questions for consideration:

1. What are the effects of ball temperature and ball inflation pressure on collision elasticity? Assume that you are a baseball coach of a poor batting team and your team is playing the best offensive team in the league. Would there be any advantage for the baseballs to be either unusually warm or cold?

2. The sweet spot of a tennis racquet can be defined in several ways. Define the three variations of the sweet spot as described by Howard Brody, tennis aficionado and physics professor at the University of Pennsylvania.

3. Do aluminum bats provide any impact advantage over wooden bats? What evidence supports your answer?